ISU-HET-99-05
UK/TP 99-09
hep-ph/9909202
September 1999

Additional Isospin-Breaking Effects in ϵ'/ϵ

S. Gardner$^{(a)*}$ **and G. Valencia**$^{(b)\dagger}$

(a) *Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506-0055*

(b) *Department of Physics and Astronomy, Iowa State University, Ames, IA 50011*

Abstract

In the analysis of ϵ'/ϵ it has been traditional to consider the isospin-breaking effects arising from electroweak-penguin contributions and from π^0-η, η' mixing, yet additional isospin-violating effects exist. In particular, we study the isospin violation which arises from the u-d quark mass difference in the hadronization of the gluonic penguin operator, engendering contributions of an effective $\Delta I = 3/2$ character. Using chiral perturbation theory and the factorization approximation for the hadronic matrix elements, we find within a specific model for the low-energy constants that we can readily accommodate an increase in ϵ'/ϵ by a factor of two.

*E-mail: gardner@pa.uky.edu
†E-mail: valencia@iastate.edu

1 Introduction

The recent measurement of a non-zero value of $\mathrm{Re}\,(\epsilon'/\epsilon)$ [1] establishes the existence of direct CP violation in $K \to \pi\pi$ decays and provides an important first check of the mechanism of CP violation in the Standard Model (SM). However, the value of $\mathrm{Re}\,(\epsilon'/\epsilon)$ which emerges from combining the recent KTeV and NA38 results [1] with the earlier NA31 and E731 results [2], yielding $\mathrm{Re}\,(\epsilon'/\epsilon) = (21.2 \pm 2.8) \cdot 10^{-4}$ [3], exceeds the "central" SM prediction of $7.0 \cdot 10^{-4}$ [4, 5] by a factor of three. This compels us to scrutinize the SM predictions in further detail: here we study isospin-violating effects arising from the u-d quark mass difference.

Isospin violation plays an important role in the analysis of ϵ'/ϵ, for the latter is predicated by the difference of the imaginary to real part ratios in the $\Delta I = 1/2$ and $\Delta I = 3/2$ $K \to \pi\pi$ amplitudes. The differing charges of the u and d quarks engender $\Delta I = 3/2$ electroweak penguin contributions, whereas π^0-η, η' mixing, driven by the u-d quark mass difference, modifies the relative contribution of the $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes in a significant way. The analyses of Refs. [6, 7, 8] consider the effect of the electromagnetic penguin operator [9] as well as of the mixing of the neutral pion with both the η [9] and the η'. Recent analyses have incorporated electroweak penguins in detail, as reviewed in Ref. [4, 10].

Here we focus on isospin-breaking effects in the gluonic penguin operator. This operator has always been described as purely $\Delta I = 1/2$ in nature, but this is only true in the limit of isospin symmetry. That is, although the short distance structure of the operator Q_6 [11], e.g., is manifestly $\Delta I = 1/2$, the differing up and down quark masses effectively distinguish the interaction of gluons with up and down quarks, so that the $\langle \pi\pi | Q_6 | K \rangle$ matrix element can possess a $\Delta I = 3/2$ component as well [12]. Alternatively, one can consider the $(8_L, 1_R)$ operators of the weak chiral Lagrangian [13, 14], which embraces operators such as a "hadronized" Q_6. In this case one finds that quark mass effects in the octet operators appear at $\mathcal{O}(p^4)$ in the weak chiral Lagrangian; this is an explicit realization of the isospin-violating effects we discuss.

In the isospin-perfect limit, ϵ'/ϵ can be written in terms of the amplitudes $A_0 \equiv A(K \to (\pi\pi)_{I=0})$ and $A_2 \equiv A(K \to (\pi\pi)_{I=2})$ as [5]

$$\frac{\epsilon'}{\epsilon} = -\frac{\omega}{\sqrt{2}|\epsilon|}\xi(1 - \Omega) \; , \tag{1}$$

where

$$\omega \equiv \frac{\mathrm{Re} A_2}{\mathrm{Re}\, A_0} \quad ; \quad \xi \equiv \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} \quad ; \quad \Omega \equiv \frac{\mathrm{Im}\, A_2}{\omega \mathrm{Im}\, A_0} \tag{2}$$

and $\omega \approx 1/22$ emerges from an analysis of $K \to \pi\pi$ branching ratios [15, 16]. The quantity ξ is driven by the gluonic penguin contribution, and a non-zero Ω reflects the presence of $\Delta I = 3/2$ contributions.

We adopt the notation $\Omega_{\rm IB}$ to denote the contribution to Ω generated by the u-d quark mass

difference[1], which parallels the original discussions of π^0-η, η' mixing effects [6, 7]. Indeed we have $\Omega_{\rm IB} = \Omega_{\eta,\eta'} + \Omega_P$, where the quantity $\Omega_{\rm P}$ is driven by isospin violation in the hadronization of the gluonic penguin and $\Omega_{\eta,\eta'}$ arises from π^0-η, η' mixing. Using the isospin decomposition [6, 7]

$$\begin{aligned} A(K^0 \to \pi^+\pi^-) &= A_0 + \frac{1}{\sqrt{2}} A_2 \\ A(K^0 \to \pi^0\pi^0) &= A_0 - \sqrt{2} A_2 \,, \end{aligned} \tag{3}$$

and introducing "$A_{\rm P}$" to denote $K \to \pi\pi$ amplitudes induced by $(8_L, 1_R)$ operators, we have

$$\Omega_{\rm IB} = \left(\frac{\sqrt{2}}{3\omega}\right) \frac{{\rm Im}\Big(A_{\rm P}(K^0 \to \pi^+\pi^-) - A_{\rm P}(K^0 \to \pi^0\pi^0)\Big)}{{\rm Im}\, A_{\rm P}(K^0 \to \pi\pi)} \tag{4}$$

with ${\rm Im}\, A_{\rm P}(K^0 \to \pi\pi) = ({\rm Im}\, A_{\rm P}(K^0 \to \pi^+\pi^-) + {\rm Im}\, A_{\rm P}(K^0 \to \pi^0\pi^0))/2$. The numerator of this expression vanishes in the absence of isospin violation. Note that a plurality of electromagnetic effects, such as final-state Coulomb rescattering in the $\pi^+\pi^-$ channel [18], can also make the right-hand side of Eq.(4) non-zero. We ignore isospin-violating electromagnetic effects all together, for they are small [18], and focus on the phenomenological consequences of the u-d quark mass difference exclusively. At leading-order in chiral perturbation theory, the weak chiral Lagrangian does not contain quark-mass-dependent effects [13] and only $\Omega_{\eta+\eta'}$ is non-zero; however, as we show below, the weak chiral Lagrangian does possess such effects in $\mathcal{O}(p^4)$.

The paper is organized as follows. In Section 2 we study the $K \to \pi\pi$ amplitudes at tree level in $\mathcal{O}(p^4)$ in the weak chiral Lagrangian. This general framework allows us to identify all the isospin breaking effects that can occur at next-to-leading order in chiral perturbation theory, albeit the low-energy constants are unknown. In Section 3 we consider the gluonic penguin operator. To estimate its contributions to the isospin breaking operators we identify in Section 2, we use the factorization approximation for the hadronic matrix elements. Within the factorization approximation, the terms of the $\mathcal{O}(p^6)$ strong chiral Lagrangian contribute to the $\mathcal{O}(p^4)$ weak chiral Lagrangian. These low-energy constants are also unknown; in this case, however, we can use a resonance-saturation model [19] to estimate them, to illustrate the effect.

2 Chiral Lagrangian Analysis

Chiral perturbation theory forms a natural framework in which to discuss isospin violation in the $K \to \pi\pi$ amplitudes. The weak chiral Lagrangian is realized in terms of the unitary matrix $U = \exp(2i\phi/f)$, which transforms under the chiral group $SU(3)_L \times SU(3)_R$ as $U \to RUL^\dagger$, where R, L are elements of $SU(3)_{R,L}$ respectively. The function ϕ represents the octet of pseudo-Goldstone

[1] We adopt this notation for simplicity and refer the reader to Ref. [17] for a detailed discussion.

bosons, where $\phi = \sum_{a=1,\ldots 8} \lambda_a \phi_a$ [14]. The chiral Lagrangian is constructed in terms of U and its derivatives, note that $L_\mu \equiv i U^\dagger D_\mu U$, as well as in terms of the function χ, which transforms as U under the chiral group. In the absence of external fields, $\chi = 2B_0 M$, where M denotes the quark mass matrix, $M = \mathrm{diag}(m_u, m_d, m_s)$, and the parameter B_0, proportional to the quark condensate $\langle \bar{q} q \rangle$, has dimensions of mass. The function χ encodes the isospin-violating effects of interest as $m_u \neq m_d$.

The $\mathcal{O}(p^2)$, CP-odd[2], weak chiral Lagrangian transforming as $(8_L, 1_R)$ under $SU(3)_L \times SU(3)_R$ has only one term [13]

$$\mathcal{L}_W^{(2)} = c_2^- \langle \lambda_7 L^2 \rangle \, , \tag{5}$$

where c_2^- is a parameter of order of the Fermi constant G_F and $\langle\rangle$ denotes a trace over flavor indices. The χ-dependent term anticipated from the form of the leading-order strong chiral Lagrangian, proportional to $\langle \lambda_7 (\chi^\dagger U + U^\dagger \chi) \rangle$, does not appear in the computation of physical amplitudes [14]. The term in question can be written as a total divergence [20] in our case and thus does not contribute, in accord with Ref. [21].

In next-to-leading order χ-dependent terms are possible. The complete $\mathcal{O}(p^4)$ weak chiral Lagrangian has been constructed in Ref. [14]. Collecting the terms which can evince isospin violation, we find [3]

$$\begin{aligned}
\mathcal{L}_{W,\mathrm{IB}}^{(4)} &= E_1^- \langle \lambda_7 \chi_+^2 \rangle + E_2^- \langle \lambda_7 \chi_+ \rangle \langle \chi_+ \rangle + E_3^- \langle \lambda_7 \chi_-^2 \rangle \\
&+ E_4^- \langle \lambda_7 \chi_- \rangle \langle \chi_- \rangle + E_5^- \langle \lambda_7 i [\chi_+, \chi_-] \rangle \\
&+ E_{10}^- \langle \lambda_7 \{\chi_+, L^2\} \rangle + E_{11}^- \langle \lambda_7 L_\mu \chi_+ L^\mu \rangle + E_{12}^- \langle \lambda_7 L_\mu \rangle \langle \{L^\mu, \chi_+\} \rangle \\
&+ E_{14}^- \langle \lambda_7 L^2 \rangle \langle \chi_+ \rangle + E_{15}^- \langle \lambda_7 i [\chi_-, L^2] \rangle \, ,
\end{aligned} \tag{6}$$

where χ_+ and χ_- are defined as

$$\begin{aligned}
\chi_+ &\equiv \chi^\dagger U + U^\dagger \chi \\
\chi_- &\equiv i(\chi^\dagger U - U^\dagger \chi) \, .
\end{aligned}$$

We can use this Lagrangian to calculate Ω_{P} from Eq. (4). Working to leading order in isospin breaking, so that merely terms linear in $m_d - m_u$ are retained, and dropping terms suppressed by M_π^2 / M_K^2, we find

$$\begin{aligned}
\Omega_{\mathrm{P}} &= \frac{2\sqrt{2}}{3\omega} \frac{M_{K^0}^2}{M_{K^0}^2 - M_\pi^2} \frac{B_0 (m_d - m_u)}{c_2^-} \Big(2E_1^- - 2E_3^- - 4E_4^- - E_{10}^- - E_{11}^- - 4E_{12}^- - E_{15}^- \Big) \\
&\approx \frac{0.12 \mathrm{GeV}^2}{c_2^-} \Big(2E_1^- - 2E_3^- - 4E_4^- - E_{10}^- - E_{11}^- - 4E_{12}^- - E_{15}^- \Big) \, .
\end{aligned} \tag{7}$$

[2] We thank G. Colangelo and J. Kambor for their generous assistence in rectifying the notational errors of our original manuscript.

[3] We choose to drop O_{13} as the one of the O_{10-14} which is not independent because it does not contribute to Eq. (4). Also, O_{32-34} are not included because they are related to O_{1-5} by equations of motion.

The terms proportional to E_1^-, E_2^-, and E_5^- can potentially generate tadpole contributions, as the eigenstates of the weak interaction are not those of mass. Consequently, we take care to remove possible tadpole contributions via the construction of Ref. [14]. Note that to $\mathcal{O}(m_d - m_u)$ and to the order in the momentum expansion to which we work, it suffices to use Eq. (5) to compute $\mathrm{Im}\, A_{\mathrm{P}}(K^0 \to \pi\pi)$. Thus merely c_2^- appears in the denominator of Eq. (7). The numerical value given reflects the use of Ref. [22] and of the relation

$$B_0(m_d - m_u) = M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2 - M_{\pi^0}^2 \tag{8}$$

which follows from the leading-order strong chiral Lagrangian, in concert with Dashen's theorem [23, 24]. The utility of Eq. (7) is limited, for the E_i^- coefficients are unknown. However, power counting in chiral perturbation theory suggests that each of the constants E_i^- is suppressed by $\mathcal{O}(\Lambda_{\chi\mathrm{SB}}^2)$ with respect to c_2^-. Thus the numerical prefactor in the last line of Eq. (7) ought determine the "natural" size of Ω_{P} — it is of order 0.1. Remarkably, these effects are of comparable numerical size to the value of Ω_η in $\mathcal{O}(p^2)$ [6, 7], so that the terms found in Eq. (7) merit further study.

For reference, it is useful to summarize the results of Refs. [6, 7] for $\Omega_{\eta+\eta'}$ in $\mathcal{O}(p^2)$ and then to proceed to enumerate *all* possible isospin-violating contributions in $\mathcal{O}(p^4)$, irrespective of whether we term them "$\Omega_{\eta+\eta'}$" or "Ω_{P}". In leading-order chiral perturbation theory, $\pi^0 - \eta$ mixing is the only $m_d \neq m_u$ effect to impact the $K \to \pi\pi$ amplitudes. Diagonalizing the neutral, non-strange meson states of the strong chiral Lagrangian in $\mathcal{O}(p^2)$, noting

$$\mathcal{L}_{\mathrm{S}}^{(2)} = \frac{f_\pi^2}{4}\Big(\langle L^\mu L_\mu \rangle + \langle \chi_+ \rangle \Big) \,, \tag{9}$$

yields the physical π^0 state in terms of the octet fields π^0 and η:

$$\left(\pi^0\right)_{\mathrm{phys}} = \pi^0 + \frac{\sqrt{3}}{4}\left(\frac{m_d - m_u}{m_s - \hat{m}}\right)\eta + \mathcal{O}(m_d - m_u)^2 \,, \tag{10}$$

where $\hat{m} = (m_d + m_u)/2$. Using Eq. (5) to compute $\mathrm{Im}\, A_{\mathrm{P}}(K \to \pi^0\eta)/\mathrm{Im}\, A_{\mathrm{P}}(K \to \pi^0\pi^0)$ yields finally [6, 7]

$$\Omega_\eta = \frac{1}{3\sqrt{2}\omega}\left(\frac{m_d - m_u}{m_s - \hat{m}}\right) \approx 0.13 \tag{11}$$

noting $(m_s - \hat{m})/(m_d - m_u) = 40.8 \pm 3.2$ [25]. The approximate equality of this result to the numerical coefficient of Eq. (7) follows as $4(M_K^2 - M_\pi^2) \approx \mathcal{O}(1\,\mathrm{GeV}^2)$. Consequently, it is also important to evaluate the impact of π^0-η mixing in $\mathcal{O}(p^4)$ on the $K \to \pi\pi$ amplitudes. This has already been done to some extent, for the usual analysis [6, 7] deviates from strict chiral perturbation theory in that an explicit η' degree of freedom appears as well, leading to both $\pi^0 - \eta'$ mixing and $\eta - \eta'$ mixing. For comparison, explicit study of π^0-η mixing in $\mathcal{O}(p^4)$, noting [24]

$$\mathcal{L}_{\mathrm{S,\,IB}}^{(4)} = L_4\langle L^2\rangle\langle\chi_+\rangle + L_5\langle L^2\chi_+\rangle + L_6\langle\chi_+\rangle^2 - L_7\langle\chi_-\rangle^2 + \frac{1}{2}L_8\langle\chi_+^2 - \chi_-^2\rangle \,, \tag{12}$$

shows that it is sensitive to the low-energy constant L_7 [24]. In an effective Lagrangian which includes the η' degree of freedom via the nonet symmetry of a large N_c approach, taking the limit of small momenta p^2 and $M_\eta^2 \ll M_{\eta'}^2$ yields an interaction of the form associated with L_7 [24]. Moreover, the η' contribution numerically saturates the value of L_7 found phenomenologically [24, 26]. The presence of the η' thus apes higher-order effects in the strong chiral Lagrangian. Including the η' as per the usual analysis [6, 7] yields[4]

$$\begin{aligned}\Omega_{\eta+\eta'} &= \Omega_\eta\Big((\cos\theta - \sqrt{2}\sin\theta)^2 + \frac{M_\eta^2 - M_\pi^2}{M_{\eta'}^2 - M_\pi^2}(\sin\theta + \sqrt{2}\cos\theta)^2\Big) \\ &\approx 2.4\,\Omega_\eta \approx 0.31 \end{aligned} \tag{13}$$

where we use $\theta = -22^\circ$ for the $\eta - \eta'$ mixing angle [7, 24]. The effect of the η' is no smaller than that of the η; this is consistent with the comparison of Eq. (7) with Eq. (11).[5] There are thus a plurality of effects which are important in $\mathcal{O}(p^4)$. Let us enumerate the possible isospin-violating effects which occur in $\mathcal{O}(m_d - m_u)$ and $\mathcal{O}(p^4)$:

i) Isospin breaking in the $\mathcal{O}(p^2)$ mass term of Eq. (9), including π^0-η mixing, acting in concert with the $\mathcal{O}(p^2)$ weak chiral Lagrangian, Eq. (5), computed to one-loop order.

ii) π^0-η mixing, realized from the $\mathcal{O}(p^2)$ mass term of Eq. (9), combined with the isospin-conserving vertices of the $\mathcal{O}(p^4)$ weak chiral Lagrangian.

iii) Next-to-leading order π^0-η mixing as per the strong chiral Lagrangian in $\mathcal{O}(p^4)$, Eq. (12), combined with the leading-order weak vertex from Eq. (5). The π^0-η' mixing effects of the usual analysis are an example of this type.

iv) Isospin violation in the vertices of the $\mathcal{O}(p^4)$ weak chiral Lagrangian, Eq. (6). This is realized as Eq. (7) and serves as our focus here, for it contains the qualitatively new effects we argue.

We wish to focus on the contribution of *iv)*, yet we cannot avoid considering that of *ii)*, for the low-energy constants E_i of Eq. (7) potentially enter here as well. Considering *exclusively* the terms of Eq.(6) we find the contribution of *ii)* to be:

$$\begin{aligned}\Omega_{\eta+\eta'}^{(4)} &= \frac{2\sqrt{2}}{3\omega}\frac{M_{K^0}^2}{M_{K^0}^2 - M_\pi^2}\frac{B_0(m_d - m_u)}{c_2^-}\Big(-2(E_3^- + E_4^- - E_5^-) + \frac{1}{2}(E_{10}^- - E_{11}^-) - 2E_{12}^- + E_{14}^- + \frac{3}{2}E_{15}^-\Big) \\ &\approx \frac{0.12\mathrm{GeV}^2}{c_2^-}\Big(-2(E_3^- + E_4^- - E_5^-) + \frac{1}{2}(E_{10}^- - E_{11}^-) - 2E_{12}^- + E_{14}^- + \frac{3}{2}E_{15}^-\Big)\,, \end{aligned} \tag{14}$$

[4]Note that using the π^0-η, η' mixing formulas resulting from the exact diagonalization of a chiral Lagrangian based on nonet symmetry in $\mathcal{O}(p^2)$ and large N_c [28] yields $\Omega_{\eta+\eta'} = 1.7\,\Omega_\eta \approx 0.22$. For comparison, note $\Omega_{\eta+\eta'} = 0.25 \pm 0.02$ from the recent analysis of Ref. [27].

[5]Large N_c arguments suggest that L_7 could dominate the low-energy constants in $\mathcal{O}(p^4)$ [24], yet this is phenomenologically not the case [24, 26].

so that there is no manifest cancellation with the terms of Eq. (7). Moreover, we find in Section 3 that the contributions of Eq. (14) are numerically smaller than those of Eq. (7) — it is the latter which contains isospin-breaking effects in the hadronization of the gluonic penguin operator. To study these effects in detail, we must turn to the factorization approximation and estimate, as in the next section, the contributions of the gluonic penguin to the E_i^- of Eq. (7).

3 Factorization

Within the context of the factorization approximation, the bosonized form of the Q_6 penguin operator appears as the product of scalar and pseudoscalar densities obtained from the strong chiral Lagrangian. The construction relevant to $K^0 \to \pi\pi$ decay is [29, 11]

$$\begin{aligned}\mathcal{L}_{\rm P} &= -\frac{G_F}{\sqrt{2}} V_{us}^\star V_{ud}\, C_6 \left(-8(\bar{s}_L q_R)(\bar{q}_R d_L)\right) \;+\; \text{h.c.} \\ &\to \frac{G_F}{\sqrt{2}} V_{us}^\star V_{ud}\, C_6\, 32 B_0^2 \frac{\delta\mathcal{L}}{\delta\chi_{3i}^\dagger}\frac{\delta\mathcal{L}}{\delta\chi_{i2}} \;+\; \text{h.c.}\;,\end{aligned} \tag{15}$$

where $q_{(L\,,R)} = (1 \mp \gamma_5)q/2$ and C_6 is defined as in Ref. [30]. Using the $\mathcal{O}(p^4)$ strong chiral Lagrangian [24], Eq. (12), one finds

$$c_2^- = \frac{G_F}{\sqrt{2}} V_{us}^\star V_{ud} \mathrm{Im}\, C_6 \left(16 B_0^2 f_\pi^2 L_5\right)\,. \tag{16}$$

Equation (15) also yields a term of the form $\langle\lambda_7(\chi_+)\rangle$, proportional to L_8, yet this is merely the weak mass term discussed earlier — it does not contribute here [20, 21]. It is well-known that the contribution of the CP-even analogue of Eq. (16) does not suffice to reproduce the phenomenological value of c_2, its associated low-energy constant [29], where we note $\mathrm{Im}\, C_6 \to \mathrm{Re}\, C_6$ in Eq. (16) yields $c_2^- \to c_2$. Equation (16) is useful nevertheless, for it serves to normalize the isospin-violating constants induced by the Q_6 operator.

The $\mathcal{O}(p^4)$ strong Lagrangian, Eq. (12), as per Eq. (15), also yields contributions to certain of the E_i^- coefficients enumerated in Eq. (6), as well as to other operators of the $\mathcal{O}(p^4)$ weak chiral Lagrangian. The non-zero contributions to E_i^- are

$$\begin{aligned} E_1^- &= E_3^- = -\,E_5^- = \frac{2L_8^2 c_2^-}{f_\pi^2 L_5} \quad;\quad E_2^- = \frac{8L_6 L_8 c_2^-}{f_\pi^2 L_5} \quad;\quad E_4^- = \frac{8L_7 L_8 c_2^-}{f_\pi^2 L_5} \\ E_{10}^- &= E_{15}^- = \frac{2L_8 c_2^-}{f_\pi^2} \quad;\quad E_{13}^- = \frac{4L_4 L_8 c_2^-}{f_\pi^2 L_5} \quad;\quad E_{14}^- = \frac{8L_6 c_2^-}{f_\pi^2}\,. \end{aligned} \tag{17}$$

Unfortunately, however, this approach does not yield a complete estimate of the coefficients of the $\mathcal{O}(p^4)$ weak chiral Lagrangian in the factorization approximation. The bosonization of Q_6, as

defined in Eq. (15), also contributes to the $\mathcal{O}(p^4)$ weak chiral Lagrangian through the $\mathcal{O}(p^6)$ strong chiral Lagrangian. Although the latter has been constructed [31, 19], its coefficients are not known, and we must turn to a model to proceed.

The use of resonance saturation allows us to estimate some of the coefficients in the $\mathcal{O}(p^6)$ strong chiral Lagrangian. This has been done for the L_i constants that appear in the $\mathcal{O}(p^4)$ strong Lagrangian [26]. In particular, the form of the terms needed in Eq. (6) suggest that scalar and pseudo-scalar resonances might be dominant. This is true for the coefficients of the $\mathcal{O}(p^4)$ strong Lagrangian that appear in Eq. (17). The constant L_7 is saturated by the η' [24], and it is reasonable to assume, in an analogous manner, that the constants L_5 and L_8 are saturated by the scalar resonances [26]. Indeed, Ref. [26] inverts this argument and uses the phenomenological values of L_5 and L_8 to fix the couplings of the scalar resonances to the pseudoscalar octet of π's and η's.

As a model for the needed $\mathcal{O}(p^6)$ counterterms, we propose the Lagrangian

$$\mathcal{L}_S = \frac{1}{2}\langle D^\mu S D_\mu S - M_S^2 S^2\rangle + c_d\langle \xi^\dagger S \xi L^2\rangle + c_m\langle \xi^\dagger S\xi\chi_+\rangle + \frac{d_m}{2}\langle \xi^\dagger S^2\xi\chi_+\rangle \tag{18}$$

for the scalar meson octet, where $U = \xi^2$. The first three terms of this Lagrangian are explicitly considered in Ref. [26]. In the limit of momenta such that $p^2 \ll M_S^2$, the scalar octet no longer plays a dynamical role and is thus "integrated out," yielding [26]

$$L_5 = \frac{c_d c_m}{M_S^2} \quad ; \quad L_8 = \frac{c_m^2}{2M_S^2} \, . \tag{19}$$

Using a scalar mass of $M_S = 0.983$ GeV and assuming that this contribution saturates $L^r_{5,8}(M_\rho)$, one finds $c_m = 0.042$ GeV, $c_d = 0.032$ GeV [26].

The last term in Eq. (18) has been recently considered in Ref. [19]. This term breaks the mass degeneracy of the states in the scalar octet, splitting the $K_0^*(1430)$ from the $a_0(980)$, for example. This term also generates some of the $\mathcal{O}(p^6)$ strong operators of interest to us. Integrating out the scalar octet one finds two terms proportional to d_m in $\mathcal{O}(p^6)$ [19]

$$\mathcal{L}_S^{(6)} = \frac{d_m c_m^2}{2M_S^4}\langle \chi_+^3\rangle + \frac{c_d c_m d_m}{M_S^4}\langle \chi_+^2 L^2\rangle \, , \tag{20}$$

which contribute to the scalar densities in the bosonization of Q_6. The new contributions are

$$\frac{E_1^-}{c_2^-} = \frac{3d_m c_m^2}{2M_S^4 L_5} \approx -4.8 \text{ GeV}^{-2} \quad ; \quad \frac{E_{10}^-}{c_2^-} = \frac{c_d c_m d_m}{M_S^4 L_5} \approx -2.4 \text{ GeV}^{-2} \, . \tag{21}$$

Within the context of our model, other contributions of the $\mathcal{O}(p^6)$ strong Lagrangian to the constants E_i are assumed to be identically zero. For the numerical estimates we fit d_m to the K_0^*-a_0 mass difference,

$$d_m = \frac{M_{K_0^*}^2 - M_{a_0}^2}{2(M_\pi^2 - M_{K^0}^2)} \sim -2.4 \, , \tag{22}$$

and use $L_5^r(\mu = M_\rho)$ of Ref. [26]. Note that the terms of Eq. (21) are numerically much larger than those of Eq. (17) — indeed, they dominate $\Omega_{\rm P}$. If we use the values of $L_{5,7,8}^r(\mu = M_\rho)$ from Ref. [26]: $L_5 = 1.4 \times 10^{-3}$, $L_7 = -0.4 \times 10^{-3}$, and $L_8 = 0.9 \times 10^{-3}$, Eq. (17) yields

$$\frac{E_1^-}{c_2^-} = 0.13 \text{ GeV}^{-2} \; ; \; \frac{E_4^-}{c_2^-} = -0.24 \text{ GeV}^{-2} \; ; \; \frac{E_{10}^-}{c_2^-} = 0.21 \text{ GeV}^{-2} \; , \tag{23}$$

where $f_\pi = 93\,\text{MeV}$. In view of the dominance of the terms computed from the $\mathcal{O}(p^6)$ coefficients, it is important to compare the relative size of the $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ coefficients induced by the scalar resonance. To illustrate, let us consider the ratio of the coefficient of the first term of Eq. (20), calling it β_1, to the coefficient L_8:

$$\frac{\beta_1}{L_8} = \frac{d_m}{M_S^2} \sim -2.4 \text{ GeV}^{-2} \; . \tag{24}$$

This ratio is large, but not inconsistent with dimensional analysis. Nevertheless, it may be naive to associate the K_0^*-a_0 mass difference with flavor-symmetry breaking as in Eq. (18). For example, quark model studies suggest that the $a_0(980)$ may well be a $K\bar{K}$ molecule [32, 33]. If we use the predicted lowest-lying isovector and strange scalar states of Ref. [33], yielding masses of 1.09 and 1.24 GeV, respectively, we find, rather, that $d_m \sim -0.76$ and that $\beta_1/L_8 \sim -0.79 \text{ GeV}^{-2}$ — this is also consistent with dimensional analysis.

We can now proceed to estimate the value of $\Omega_{\rm P}$ using our estimated low-energy constants. Were we merely to use the numbers of Eq. (21) and $d_m = -2.4$ we would obtain

$$\Omega_{\rm P} = \left(0.12\,\text{GeV}^2\right) \frac{d_m c_m (3c_m - c_d)}{M_S^4 L_5} \sim -0.85 \; . \tag{25}$$

This unexpectedly large result is driven by the value of d_m found in Eq. (22): using $d_m \sim -0.76$ yields $\Omega_{\rm P} \sim -0.28$. The sign of d_m and thus of $\Omega_{\rm P}$ in our picture is the consequence of the mass of lowest-lying strange scalar being greater than that of the lowest-lying isovector scalar. Collecting the contributions of Eq. (21) and Eq. (23) yields

$$\Omega_{\rm P} = -0.79\,(-0.21) \tag{26}$$

for $d_m = -2.4\,(-0.76)$. Note that Eq. (14) and $L_6^r(\mu = M_\rho) = 0.2 \cdot 10^{-3}$ [26] yields $\Omega_{\eta+\eta'}^{(4)} = -0.12\,(-0.03)$, so that writing $\Omega_{\rm IB}^{(4)} = \Omega_{\rm P} + \Omega_{\eta+\eta'}^{(4)}$ yields $\Omega_{\rm IB}^{(4)} = -0.91\,(-0.24)$. For reference, the value of Eq. (4) used in the "central value" of ϵ'/ϵ in Ref. [4] is $\Omega_{\eta+\eta'} = 0.25 \pm 0.05$, whereas that used in Ref. [10] is $\Omega_{\eta+\eta'} = 0.25 \pm 0.10$. The changes in $\Omega_{\rm IB}$ found in $\mathcal{O}(p^4)$ impact ϵ'/ϵ in a significant manner. Using the simple formula of Eq. (1.7) in Ref. [4] shows that under $\Omega_{\rm IB} = 0.25 \to -0.25$ the value of ϵ'/ϵ increases by a factor of 2.2. Thus a very small or negative value of $\Omega_{\rm IB}$ generates an increase in ϵ'/ϵ with respect to the usual value cited [4]. It is particularly noteworthy that the range in our estimates of $\Omega_{\rm IB}^{(4)}$ *exceed* the central value of $\Omega_{\eta+\eta'}$ used in Refs. [4, 10]. The detailed numerical results we find do rely on a simple model; nevertheless, a substantial increase in the error associated with $\Omega_{\rm IB}$, Eq. (4), is in order.

4 Conclusions

We have shown that there are isospin-breaking effects in ϵ' which have not been previously considered. Specifically, we have examined the role of isospin violation in the matrix elements of the gluonic penguin operator within the context of chiral perturbation theory. Although the presence of unknown low-energy constants implies that we lack a reliable way to calculate these effects, we believe such limitations underscore the need for a larger uncertainty in the theoretical value of ϵ'/ϵ than currently in vogue. In particular, the recent reviews of Refs. [4, 10] use 0.25 ± 0.05 and 0.25 ± 0.10, respectively, for the value of $\Omega_{\rm IB} = \Omega_{\eta+\eta'}$. Our estimate of $\Omega_{\rm IB}$ from the specific $m_d \neq m_u$ effects we consider ranges from $0.1 \rightarrow -0.7$. This range reflects a variation in ϵ'/ϵ of more than a factor of two.

Acknowledgments The work of S.G. and G.V. is supported in part by the DOE under contract numbers DE-FG02-96ER40989 and DE-FG02-92ER40730, respectively. We thank J. F. Donoghue, C.-J. David Lin, and M. B. Wise for useful conversations, and we thank J. Bijnens for pointing out Ref. [19]. We are grateful to the Institute for Nuclear Theory at the University of Washington, the SLAC Theory Group, the Summer Visitor's Program of the Fermilab Theory Group, and M. B. Wise for hospitality during the completion of this work.

References

[1] A. Alavi-Harati *et al.* (KTeV Collaboration), *Phys. Rev. Lett.* **83** (1999) 22; P. Debu for the NA48 Collaboration, http://www.cern.ch/NA48/FirstResult/slides.html

[2] G.D. Barr *et al.* (NA31 Collaboration), *Phys. Lett.* **317B** (1993) 233; L.K. Gibbons *et al.* (E731 Collaboration), *Phys. Rev. Lett.* **70** (1993) 1203.

[3] http://www.cern.ch/NA48/FirstResult/slide20.html

[4] S. Bosch *et al.*, hep-ph/9904408.

[5] A. Buras, hep-ph/9806471, to appear in *Probing the Standard Model of Particle Physics*, F. David and R. Gupta, eds. (Elsevier Science B. V., Amsterdam, 1998).

[6] J. F. Donoghue *et al.*, *Phys. Lett.* **179B** (1986) 361.

[7] A. J. Buras and J. M. Gerard, *Phys. Lett.* **192B** (1987) 156.

[8] H.-Y. Cheng, *Phys. Lett.* **B201** (1988) 155; M. Lusignoli, *Nucl. Phys.* **B325** (1989) 33.

[9] J. Bijnens and M. Wise, *Phys. Lett.* **137B** (1984) 245.

[10] S. Bertolini, M. Fabbrichesi, and J. Eeg, hep-ph/9802405.

[11] E. de Rafael, TASI 94 lectures, hep-ph/9502254 and references therein.

[12] This prospect was first discussed in the context of $B \to \pi\pi$ decays, see S. Gardner, *Phys. Rev.* **D59** (1999) 077502.

[13] J. A. Cronin, *Phys. Rev.* **161** (1967) 1483.

[14] J. Kambor, J. Missimer, and D. Wyler, *Nucl. Phys.* **B346** (1990) 17.

[15] T. J. Devlin and J. O. Dickey, *Rev. Mod. Phys.* **51** (1979) 237.

[16] J. Kambor, J. Missimer, and D. Wyler, *Phys. Lett.* **261** (1991) 496.

[17] S. Gardner and G. Valencia, in preparation.

[18] V. Cirigliano, J. F. Donoghue, and E. Golowich, hep-ph/9810488; hep-ph/9907341.

[19] G. Amorós, J. Bijnens, and P. Talavera, hep-ph/9907264.

[20] C. Bernard *et al.*, *Phys. Rev.* **D32** (1985) 2343.

[21] G. Feinberg, P. Kabir, and S. Weinberg , *Phys. Rev. Lett.* **3** (1959) 527.

[22] C. Caso *et al.*, *Eur. Phys. J.* **C 3** (1998) 1.

[23] R. Dashen, *Phys. Rev.* **183** (1969) 1245.

[24] J. Gasser and H. Leutwyler, *Nucl. Phys.* **B250** (1985) 465.

[25] H. Leutwyler, *Phys. Lett.* **B378** (1996) 313.

[26] G. Ecker *et al.*, *Nucl. Phys.* **B321** (1989) 311.

[27] H.-Y. Cheng, hep-ph/9906403.

[28] H. Leutwyler, *Phys. Lett.* **B374** (1996) 181.

[29] S. Chivukula, J. Flynn, and H. Georgi, *Phys. Lett.* **B171** (1986) 453.

[30] G. Buchalla, A. J. Buras, and M. K. Harlander, *Nucl.Phys.* **B337** (1990) 313.

[31] H. W. Fearing and S. Scherer, *Phys. Rev.* **D53** (1996) 315; J. Bijnens, G. Colangelo, and G. Ecker, hep-ph/9902437.

[32] J. Weinstein and N. Isgur, *Phys. Rev. Lett.* **48** (1982) 659; *Phys. Rev.* **D27** (1983) 588.

[33] S. Godfrey and N. Isgur, *Phys. Rev.* **D32** (1985) 189.